**CS 404 – Artificial Intelligence**

**HW 4 – 2020 Spring**

150pt

**Objective:** Learning about Propositional Logic

**1) 20 pt - Decide whether each of the following sentences is valid, unsatisfiable, or satisfiable** (not valid, but only satisfiable). **Show how you come to that decision using truth table enumeration or logical equivalence rules.**

* Smoke => Smoke ………………………………………… 10pt

Smoke Smoke=>Smoke

T T

F T

**Circle the correct answer: Valid,** **Satisfiable, Unsatisfiable**

* Smoke => Fire ………………………………………….. 10pt

Smoke Fire Smoke => Fire

T T T

F T T

T F F

F F T

**Circle the correct answer: Valid, Satisfiable, Unsatisfiable**

**2) 40pt – Truth Table Enumeration for Inference**

Assume we have the knowledge base **KB: Rain ∧ (Rain ⇒ Wet) ∧ (Snow ⇒ Cold)** and the given propositions are the only ones in the KB.

1. **25 pts – How many *possible worlds*** (truth value assignments to the propositions) **are *models* of the KB?** Show your work by filling the truth table for the KB.

Rain Wet **Rain ∧ (Rain ⇒ Wet)**

T T T ^ T=>T ~ T

T F T ^ T=>F ~ F

F T F ^ F=>T ~ F

F F F ^ F =>F ~ F

When Rain is true in order to make Rain=>Wet,there is only one option which is to make Wet true.

No other possible action.

In order to make Snow => Cold we know that a truth table of implies cover :

**Snow** **Cold** **(Snow ⇒ Cold) Rain ∧ (Rain ⇒ Wet) ∧ (Snow ⇒ Cold)**

T T T => T ~ T T

T F T => F ~ F F

F T F => T ~ T T

F F F => F ~ T T

If it is Snow it must be Cold.If it is not Snow it is Cold(Which makes sense since it is raining for sure.)

If it is not Snow and not Cold.(Which also makes sense since it can still be raining but not that cold in order to Snow.)

So it is known that Rain must be True else it this logic won’t result with True.If rain is True Wet must be True else this logic won’t result True.Rain and Wet are actually independent variables.In order to know the number of possible worlds **(Snow ⇒ Cold)** must be True and since only this part creates a dependency.There are 3 possible scenarios where this case would result True.

1. **15pts** – **Extend the above truth table** (use truth table enumeration method) to show whether the knowledge base entails **α=Wet.**

**State your answer here:** Entails/ Does not Entail because every time KB is true,the Wet must also be true.In another saying , in order to make the KB true Wet must always be true.

Rain Wet Snow Cold **Rain ∧ (Rain ⇒ Wet) ∧ (Snow ⇒ Cold)**

T T T T T

T T T F False since Snow=>Cold cannot be true while Snow is true Cold false

T T F T T

T F T T

F T T T

F F T T

T F F T

T T F F T

F T F T

F T T F

T F T F

T F F F

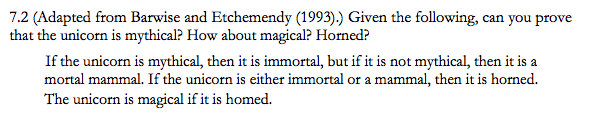
F F F T

F F T F

F T F F

F F F F

**3) 90pt – AIMA 3rd Ed. Q. 7.2**

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1. **20pt** – **First state the English paragraph as a set of Propositional Logic sentences.** **Please use the given two-letter proposition names below.** Be careful about what propositions to use (especially for the the statement “mortal mammal”), considering all of the paragraph.

unicorn =>mythical=>immortal

unicorn=>!mythical=>!immortal ^ mammal

unicorn=>(immortal || mammal)=>horned

unicorn=>horned=>magical

P1.mythical=>immortal

P2.!mythical=>!immortal ^ mammal

P3.(immortal v mammal)=>horned

P4.horned=>magical

Lets first take the CONTRAPOSITIVE of P1 which results as !immortal=>!mythical (1).Then take 1 and make HYPOTHETICAL SYLLOGISM with P2 which results as !immortal=>!immortal ^ mammal (2).

Then take 2 and apply IMPLICATION DEFN which results as immortal v !immortal ^ mammal (3).Call the DISTRIBUTIVE of 3 which results as (immortal v !immortal) ^ (immortal ^ mammal) (4).The first premises causes True automatically True ^ (immortal v mammal) which can be seen that the result is dependent on right side of the proposition (immortal v mammal) due to TAUTOLOGY combined with IDENTITY (5).

Modus ponens of P3 using 5 which is (immortal v mammal)=>(immortal v mammal)=>horned which results as horned (6).By using MODUS PONENS again with P4 and 6 as horned=>horned=>magical which results as magical (7).

The unicorn is “horned” and “magical” but there is no propositional logic that shows the unicorn is “mythical”.

1. **20pt** – **Convert the above KB into Conjunctive Normal Form.** Show your work clearly.

P1.mythical=>immortal

P^1. !mythical v immortal

P2.!mythical=>!immortal ^ mammal

P^2. mythical v (!immortal ^ mammal)

P^2. (mythical v !immortal) ^ (!immortal v mammal)

P3.(immortal v mammal)=>horned

P^3. !(immortal v mammal) v horned

P^3. (!immortal ^ !mammal) v horned

P^3. (!immortal v horned) ^ (!mammal v horned)

P4.horned=>magical

P^4. !horned v magical

1. **50pt** **– See if the KB entails each of the following conclusions: “unicorn is mythical”, “unicorn is magical”, “unicorn is horned”.** Use the indicated method, if available.

For each of them, **please indicate the inference method you are using and clearly indicate your conclusion at the beginning:**

1. **10pts - “unicorn is mythical”:**

**Conclusion**: Using all possible options, we can show

that we can conclude?/**not conclude**? (infer/**not infer**?) that the unicorn is mythical.

**Show your work or state your argument:In the prev.1 part of the question we have seen that by using Propositional Logic sentences it is stated that it is not possible to show this conclusion.**

1. **30pt - “unicorn is horned”:**

**Use either:**

**- simple resolution:** that is apply resolution many imes to see whether you can infr Horned, or

**- resolution refutation** (aka proof by contradiction):that is, add the negated form of whatever you want to check for entailment (e.g. ¬Horned), and see if you can reach a contradiction. This would show that the KB+negative is unsatisfiable, hence the conclusion can be inferred).

(immortal v mammal)=>(immortal v mammal)=>horned

!(immortal v mammal)v(immortal v mammal)=>horned

!( !(immortal v mammal)v(immortal v mammal)) v horned

( !(immortal v mammal)^(immortal v mammal)) v horned

( !(immortal v mammal) ^ horned) v (immortal v mammal) ^ horned)

A = (immortal v mammal)

!A = ! (immortal v mammal)

(A ^ horned) v (!A ^ horned) ⇔ horned ^ (A v !A)

The unicorn is horned.This case entails since applies all A=Horned.

1. **10pt - “unicorn is magical”:**

From previous part we know that horned =>magical

horned => (horned=>magical)

**Use Modus Ponens method and state your conclusion.**